PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Surface Reflexion of Earthquake Waves

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VII. *Surface Reflexion of Earthquake Waves.*

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[PLATES 3 AND 4.]

A SEISMOGRAM, or instrumental time record of the earth movement produced by an earthquake at some distance, may be divided into three main parts, known as the first phase, the second phase, and the long-wave phase respectively. As a rule, the experienced observer has little difficulty in distinguishing these phases, and although difficult cases do arise, there is general agreement that the three divisions are characteristic and correspond to definite properties of the earth.

The beginning of the first phase is denoted by P and the beginning of the second phase by S, and these are interpreted as representing the arrival of longitudinal and transversal waves which start simultaneously at the focus of the earthquake and reach the observing station at different times. By general convention P and S are also used to represent the time interval between the occurrence of the earthquake and the arrival of the corresponding waves at the observing station.

Time curves giving the values of P and S as functions of the epicentral distance were obtained by ZÖPPRITZ in 1907 ('Göttinger Nachrichten'). The tables of values are in general use at seismological observatories. They have proved remarkably successful in determining epicentres of earthquakes, and the theoretical analysis by WIECHERT has led to inferences about the interior of the earth which are of the greatest interest. Doubtless as observational data improve these time curves may have to be modified, but only by way of correction and not as regards the main broad features.

In addition to the primary features called P and S, a seismogram may show well-marked sharp impulses, and in WIECHERT'S opinion these represent the arrival of waves that have undergone one or more reflexions at the earth's surface. On this view PR_n would denote a longitudinal wave that has been reflected n times at the surface, thus dividing the epicentral distance into $(n+1)$ equal parts, and a similar notation SR_n would apply to transversal waves. We may also contemplate waves P_nS_m that have undergone change from longitudinal to transversal (or *vice versâ*) on reflexion.

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There is no doubt that observation supports the view that these effects actually occur. But there are difficulties. The times of arrival are not always in agreement with theoretical calculation from the curves for P and S, and the relative magnitudes of the reflected waves to P and S undergo changes at different distances that are difficult to understand. Thus it is doubtful if the reflexions are shown at all at less than 3000 km.; at 8000 km. PR_1 is about as large as P, while at 12,000 km. PR_1 is substantially larger than P.

The present investigation was undertaken with a view to throwing some light on these peculiar phenomena. Owing largely, no doubt, to the circumstance that most observatories possess instruments for recording only the horizontal components (some have only one), discussion has in many cases been confined to the phenomena shown in that component. The introduction of a really reliable apparatus for the vertical component by Prince GALITZIN, about seven years ago, has led to results of great value, and has shown how important it is to study the vertical and horizontal components in conjunction.

Analysis of the time curves for P and S led WIECHERT and ZÖPPRITZ to the conclusion that the corresponding seismic rays do not travel in straight lines from the focus, but that they dip down into the earth as if the speed of propagation increased with the depth. The results of their investigation will be found in GALITZIN'S 'Lectures on Seismometry,' or my own monograph on 'Modern Seismology.'

The surface values are 7·17 km. per second for P and 4·01 km. per second for S. These are substantially higher than the values found for surface rocks, and emphasize the fact that in using the term surface we do, on this view, imagine the heterogeneous crust of the earth replaced by a purely ideal surface at which the speeds acquire the values stated. The justification of this assumption and the determination of the true effect of the crust will have to be undertaken before long.

We have now to consider the magnitude of the ground movement experienced when a wave is incident on the surface. This problem has been dealt with by KNOTT ('Phil. Mag.,' 1899) and WIECHERT ('Gött. Nach.,' 1907), but only partially. We shall assume that at reflexion the waves may be treated as plane and the surface as plane.

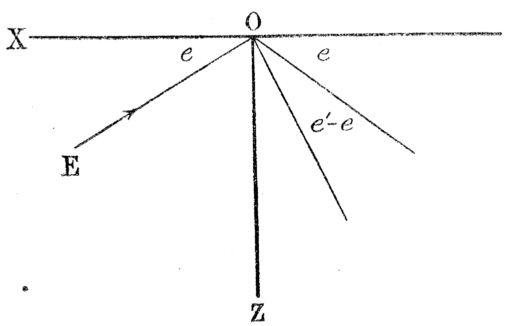

Fig. 1.

In the figure (1) let OX represent the intersection of the earth's surface supposed plane with the plane of the paper and let OZ be drawn vertically downwards. The lower side of OX represents the earth, in which the speeds of propagation of longitudinal and transversal waves are V_1 and V_2 at the surface. The upper side of OX is a vacuum in which no waves are propagated.

Let a ray EO be incident at the point O, making an angle e with OX. This angle is usually called the angle of emergence. The term is misleading. It is really the co-angle of incidence in the optical analogy, and when occasion arises I propose to call it the angle of impingence.

(1) Let the incident wave be longitudinal. The component displacements ξ_1 and ζ_1 are

$$(\xi_1, \zeta_1) = -\mathrm{A}(\cos e, \sin e) f\{t+(x \cos e+z \sin e)/\mathrm{V}_1\},$$

where f is any arbitrary function.

This wave gives rise to a reflected longitudinal disturbance making an equal angle e on the other side of OZ, expressed by

$$(\xi_2, \zeta_2) = -\mathrm{A}_2(\cos e, -\sin e) f\{t+(x \cos e-z \sin e)/\mathrm{V}_1\},$$

and a reflected transversal disturbance making an angle e' on the other side of OZ, expressed by

$$(\xi_3, \zeta_3) = \mathrm{A}_3(\sin e', \cos e') f\{t+(x \cos e'-z \sin e')/\mathrm{V}_2\}.$$

At the surface of separation OX the stresses must vanish, and this requires that

$$\frac{\partial \xi}{\partial z}+\frac{\partial \zeta}{\partial x} = 0$$

and

$$(\mathrm{V}_1^2-2\mathrm{V}_2^2)\left(\frac{\partial \xi}{\partial x}+\frac{\partial \zeta}{\partial z}\right)+2\mathrm{V}_2^2\frac{\partial \zeta}{\partial z} = 0,$$

when $z = 0$.

Thus we get the relations

$$\mathrm{A}-\mathrm{A}_2 = \mu \mathrm{A}_3 \cos 2e'/\sin 2e,$$

$$\mathrm{A}+\mathrm{A}_2 = \mu^{-1}\mathrm{A}_3 \sin 2e'/\cos 2e',$$

where

$$\mu = \mathrm{V}_1/\mathrm{V}_2 \qquad \text{and} \qquad \mu \cos e' = \cos e.$$

It may be noted that

$$\sin 2e\,(\mathrm{A}^2-\mathrm{A}_2^2) = \sin 2e'\,\mathrm{A}_3^2,$$

which satisfies the energy condition that the rate of arrival of energy by the incident waves is equal to the rate at which energy passes away by the reflected waves.

Solving the above equations, we get

$$\mathrm{A}_2/\mathrm{A} = \frac{\{\sin 2e \sin 2e'-\mu^2 \cos^2 2e'\}}{\{\sin 2e \sin 2e'+\mu^2 \cos^2 2e'\}},$$

$$\mathrm{A}_3/\mathrm{A} = \frac{2\mu \sin 2e \cos 2e'}{\{\sin 2e \sin 2e'+\mu^2 \cos^2 2e'\}}.$$

The resultant horizontal movement along OX is H, where

$$H = -(A+A_2)\cos e + A_3 \sin e',$$

thus

$$H/A = -\frac{2\mu^2 \sin e \sin 2e'}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e'\}}.$$

The resultant vertical movement along OZ is V, where

$$V = -(A-A_2)\sin e + A_3 \cos e'$$

thus

$$V/A = \frac{2\mu^2 \sin e \cos 2e'}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e'\}}.$$

The apparent angle of emergence $\bar{e}$ is given by

$$\tan \bar{e} = V/H = -\cot 2e',$$

and hence

$$\sin \bar{e} = 1 - 2\mu^{-2} \cos^2 e,$$

a relation obtained by WIECHERT.

TABLE I.—Longitudinal Disturbance Incident.

$\mu = 1{\cdot}788, \qquad \mu \cos e' = \cos e.$

e.	e'.	A_2/A.	A_3/A.	H/A.	V/A.
°	° ′				
0	56 0	−1·0000	−0·0000	−0·0000	−0·0000
5	56 8	0·4814	0·3797	0·8322	0·3407
10	56 35	0·2229	0·5946	1·2592	0·5398
15	57 18	0·0985	0·7380	1·4918	0·6830
20	58 18	0·0544	0·8466	1·6084	0·8054
25	59 33	0·0609	0·9346	1·6563	0·9219
30	61 2	0·1023	1·0039	1·6570	1·0381
35	62 44	0·1689	1·0587	1·6219	1·1555
40	64 38	0·2537	1·0909	1·5574	1·2732
45	66 42	0·3501	1·0989	1·4692	1·3894
50	68 56	0·4537	1·0797	1·3585	1·5016
55	71 17	0·5584	1·0315	1·2307	1·6077
60	73 46	0·6607	0·9534	1·0846	1·7047
65	76 20	0·7553	0·8465	0·9256	1·7908
70	78 58	0·8383	0·7131	0·7556	1·8640
75	81 41	0·9070	0·5566	0·5743	1·9225
80	84 26	0·9580	0·3817	0·3867	1·9651
85	87 12	0·9894	0·1941	0·1952	1·9913
90	90 0	1·0000	0·0000	0·0000	2·0000

In general A_2 vanishes for two values of e, and in the particular case $\mu^2 = 3$, which holds for an ideal material with POISSON'S ratio $= \frac{1}{4}$, the values at which A_2 vanishes are $e = 12° \ 47'$ and $e = 30°$.

If, however, we take the surface values given by ZÖPPRITZ, viz., $V_1 = 7{\cdot}17$ km. per second, $V_2 = 4{\cdot}01$ km. per second, we have $\mu = 1{\cdot}788$, and A_2 does not vanish for any real value of e, but falls to a very small minimum about $e = 20°$.

The numerical values of A_2, A_3, H, and V for different angles of impingence are given in Table I., and the results are also shown graphically in fig. 2, wherein A is taken as numerically -1.

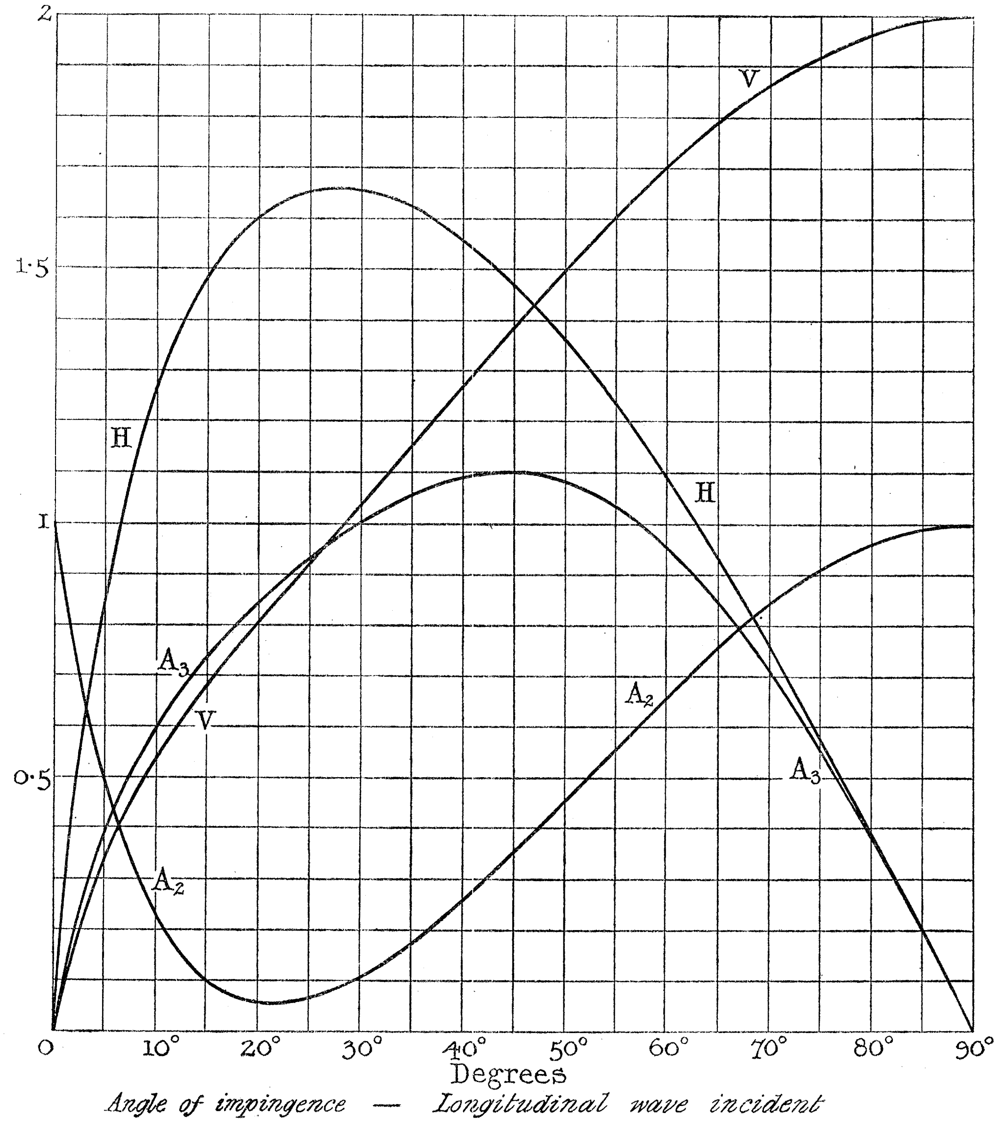

Fig. 2.

(2) If the incident wave is transversal and the vibration at right angles to the plane of the paper there is no reflected longitudinal wave for any angle of incidence, and the reflected transversal wave at an angle e is equal in magnitude to the incident wave. The resultant motion of the ground is entirely horizontal at right angles to

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OX and precisely twice the magnitude of the incident disturbance for all angles of incidence.

(3) If the incident wave is transversal and the vibration in the plane of the paper we may assume that the incident disturbance is represented by

$$(\xi_1, \zeta_1) = A(-\sin e, \cos e) f\{t+(x \cos e+z \sin e)/V_2\}.$$

This will give rise to a reflected transversal disturbance at an angle e on the other side of OZ, expressed by

$$(\xi_2, \zeta_2) = A_2(\sin e, \cos e) f\{t+(x \cos e-z \sin e)/V_2\},$$

and a reflected longitudinal disturbance at an angle e' on the other side of OZ, expressed by

$$(\xi_3, \zeta_3) = A_3(-\cos e', \sin e') f\{t+(x \cos e'-z \sin e')/V_1\}.$$

The vanishing of the stresses at the surface of separation leads to the relations

$$A-A_2 = -\mu A_3 \cos 2e/\sin 2e,$$

$$A+A_2 = -\mu^{-1} A_3 \sin 2e'/\cos 2e,$$

where

$$\mu = V_1/V_2 \qquad \text{and} \qquad \mu \cos e = \cos e'.$$

We note that the energy condition is satisfied by these equations.

Solving the equations, we get

$$A_2/A = \frac{\{\sin 2e \sin 2e' - \mu^2 \cos^2 2e\}}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e\}},$$

$$A_3/A = -\frac{2\mu \sin 2e \cos 2e}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e\}}.$$

The resulting horizontal motion of the ground is H, where

$$H/A = \frac{2\mu^2 \sin e \cos 2e}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e\}},$$

and the resulting vertical motion is V, where

$$V/A = \frac{2\mu \sin e' \sin 2e}{\{\sin 2e \sin 2e' + \mu^2 \cos^2 2e\}}.$$

For the ideal case $\mu^2 = 3$, A_2 would vanish for $e = 55° \, 44'$ and $e = 60°$; but in our actual case $\mu = 1{\cdot}788$, A_2 does not vanish but falls to a small minimum near $e = 58°$. In the present case there is no real value of e' until e attains the value 56°. Thus in the range from $e = 0°$ to $e = 56°$ there is no reflected longitudinal disturbance, but there is a type of disturbance practically confined to the surface of separation and a reflected transversal wave differing in phase from the incident wave. Within this range the values of A_2, A_3, H and V are complex, except at the special point $e = 45°$,

TABLE II.—Transversal Wave Incident.

$\mu = 1{\cdot}788, \qquad \mu \cos e = \cos e'.$

Angles Less than the Critical Angle 56°.

e.	Mod A_2/A.	Mod A_3/A.	Mod H/A.	Mod V/A.
°				
0	1·0000	0·0000	0·0000	0·0000
5	1·0000	0·3084	0·2768	0·4587
10	1·0000	0·4744	0·4306	0·7316
15	1·0000	0·5371	0·4971	0·8733
20	1·0000	0·5465	0·5199	0·9632
25	1·0000	0·5228	0·5157	1·0371
30	1·0000	0·4700	0·4852	1·1114
35	1·0000	0·3820	0·4169	1·1953
40	1·0000	0·2401	0·2802	1·2942
45	1·0000	0·0000	0·0000	1·4142
50	1·0000	0·4635	0·6446	1·5120
55	1·0000	1·5389	2·3986	0·3726

Angles e from 56° to 90°.

e.	e′.	A_2/A.	A_3/A.	H/A.	V/A.
° ′	′				
56 0	0	−1·0000	+2·7685	−4·4261	+0·0000
56 8	5	0·4814	2·0235	3·2462	0·4654
56 35	10	0·2229	1·5983	2·5946	0·7043
57 18	15	0·0985	1·3419	2·2206	0·8343
58 18	20	0·0544	1·1776	2·0035	0·8996
59 33	25	0·0609	1·0660	1·8806	0·9263
61 2	30	0·1023	0·9841	1·8165	0·9267
62 44	35	0·1689	0·9176	1·7907	0·9071
64 38	40	0·2537	0·8577	1·7898	0·8710
66 42	45	0·3501	0·7985	1·8047	0·8217
68 56	50	0·4537	0·7355	1·8292	0·7598
71 17	55	0·5584	0·6672	1·8588	0·6883
73 46	60	0·6607	0·5910	1·8899	0·6066
76 20	65	0·7553	0·5074	1·9200	0·5177
78 58	70	0·8383	0·4168	1·9469	0·4226
81 41	75	0·9070	0·3186	1·9694	0·3212
84 26	80	0·9580	0·2155	1·9861	0·2163
87 12	85	0·9894	0·1091	1·9966	0·1092
90 0	90	1·0000	0·0000	2·0000	0·0000

for which they become real. For e, between 56° and 90°, the values are all real. We thus divide Table II. giving numerical values into the range $e = 0°$ to 56°, for

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which the "modulus" is given, and the range $e = 56°$ to $90°$, for which the values are real and for which there is a real reflected longitudinal wave as well as a real reflected transversal wave.

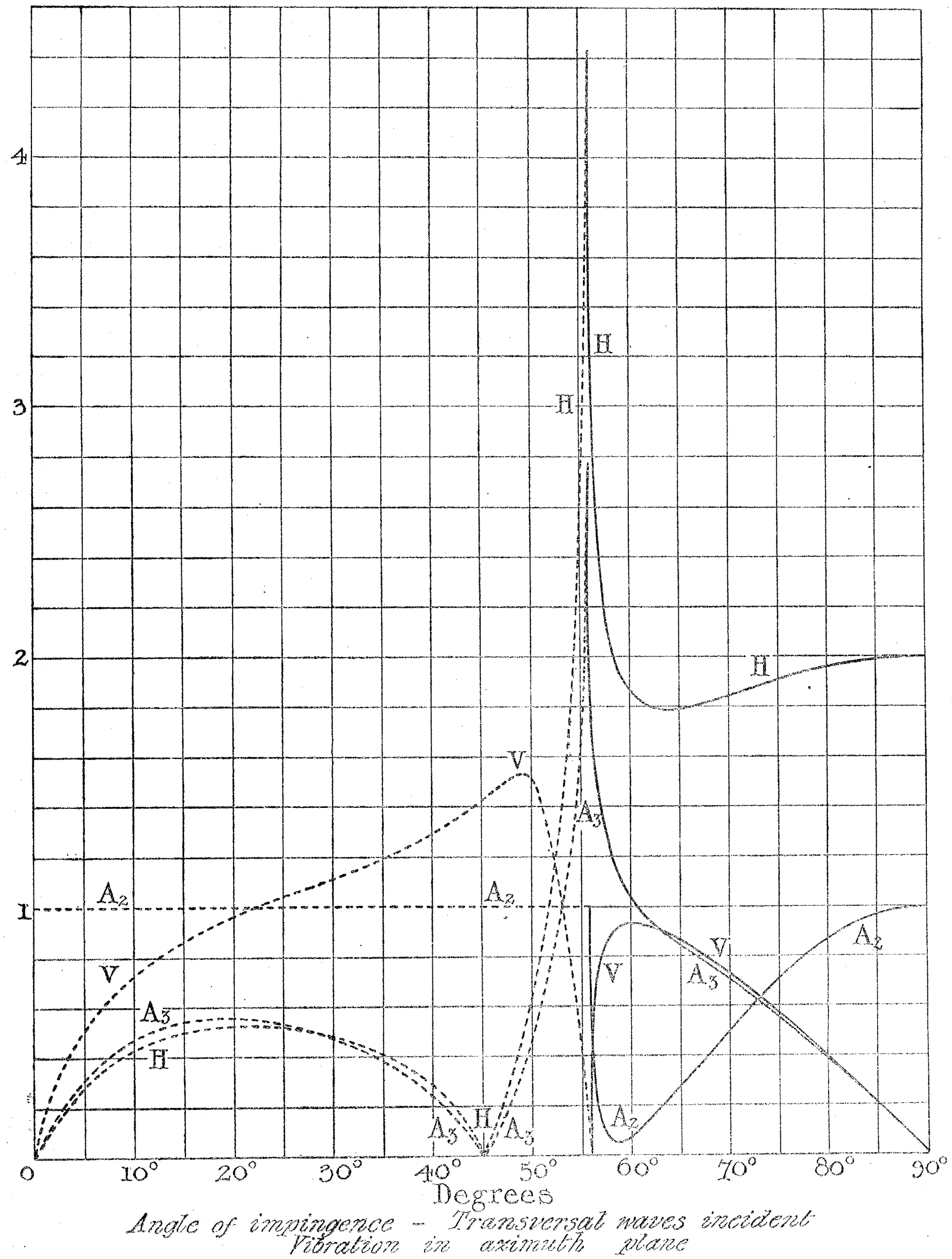


Fig. 3.

These results are exhibited graphically in fig. 3, wherein A has been taken as numerically 1, and for convenience, the – sign has been disregarded.

It is important to remember that the results we have obtained for reflexion of elastic disturbance are not confined to periodic waves (*cf.* Lord KELVIN, 'Phil. Mag.,' 1899). The function we have denoted by f may be any arbitrary function provided it represents a disturbance that can be propagated into an undisturbed region (see LOVE, 'Theory of Elasticity,' p. 283). Thus as long as A_2 and A_3 remain real precisely the same functional form represents the incident and reflected disturbances. This is the case for longitudinal disturbance impingent at all angles e from 0° to 90°, for transversal disturbance (vibration horizontal) at all angles e from 0° to 90°, and for transversal disturbance (vibration vertical) at the particular angle $e = 45°$ and from $e = \sec^{-1}\mu$ (about 56°) to 90°. But when A_2 and A_3 becomes complex, as in the case of transversal waves (vibration vertical) for angles e between 0° and $\sec^{-1}\mu$, we must start by taking a complex form for f and finally select the real parts from the various expressions in the manner familiar in optical theory.

Let $f(\theta)$ be the incident transversal disturbance where

$$\theta = t + (x \cos e + z \sin e)/V_2,$$

then the Fourier resolution of $f(\theta)$ gives

$$f(\theta) = \pi^{-1}\int_{-\infty}^{+\infty} f(\lambda)\, d\lambda \int_0^\infty \cos \alpha\,(\theta - \lambda)\, d\alpha.$$

Since in the reflected transversal disturbance

$$A_2/A = e^{i\phi_2},$$

the real part of the corresponding disturbance is

$$\pi^{-1}\int_{-\infty}^{+\infty} f(\lambda)\, d\lambda \int_0^\infty \cos\{\alpha\,(\theta_2 - \lambda) + \phi_2\}\, d\alpha,$$

where

$$\theta_2 = t + (x \cos e - z \sin e)/V_2.$$

We also have

$$A_3/A = M_3 e^{i\phi_3}.$$

There is no real angle of reflexion of the longitudinal disturbance, but if we write

$$\theta_3 = t + x \cos e/V_1 \qquad \text{and} \qquad \cosh\psi = \mu \cos e,$$

the longitudinal disturbance is expressed by

$$\xi_3 = \pi^{-1}\int_{-\infty}^{+\infty} f(\lambda)\, d\lambda \int_0^\infty M_3 \cosh\psi e^{-V_1^{-1}\alpha z \sinh\psi} \cos\{\alpha\,(\theta_3 - \lambda) + \phi_3\}\, d\alpha,$$

$$\zeta_3 = \pi^{-1}\int_{-\infty}^{+\infty} f(\lambda)\, d\lambda \int_0^\infty M_3 \sinh\psi e^{-V_1^{-1}\alpha z \sinh\psi} \sin\{\alpha\,(\theta_3 - \lambda) + \phi_3\}\, d\alpha.$$

The character of the reflected transversal disturbance will depend on the precise form of $f(\lambda)$, and, in general, the reflected disturbance will differ from the incident disturbance. There will be a "trail."

We have not introduced dispersion, but if this exists (as most probably it does) it will give rise to a "trail" with all classes of disturbance, not confined to the special range we have just considered, in a manner analogous to a pressure disturbance travelling over deep water (see LAMB, 'Hydrodynamics').

The angle of impingence may be calculated from the time curve by means of the formula due to WIECHERT,

$$\cos e = V_0\, dT/d\Delta,$$

where $dT/d\Delta$ is the slope of the time curve at epicentral distance Δ and V_0 is the surface value of the speed of the corresponding wave whether longitudinal or transversal. The results are given in Table III. along with the computed value of $\bar{e}$ for longitudinal waves and the value as directly observed at Pulkovo by measurement of the horizontal and vertical disturbance.

We now consider the magnitude of the first impulse P. Fig. 2 shows that for a given value of the incident longitudinal disturbance the horizontal motion of the ground exceeds the vertical until the angle of impingence is about 47°, where the two become equal. For greater values of e the vertical motion exceeds the horizontal. Further, H attains its maximum for $e = 27°$ and thereafter diminishes, which V continues to increase right up to 90°.

Thus quite apart from the falling off of amplitude with distance on account of spherical divergence, we should expect H to diminish in importance as we pass to great distances, while V, being relatively small for small distances, becomes equal to H for Δ about 2800 km., and for greater distances V exceeds H, the ratio being about 2 at 7000 km. and still greater as the distance increases.

This is in general agreement with observation both at Pulkovo and Eskdalemuir, where it has been observed that at about 2500 km. V is about the same magnitude as H, while at 8000 km. or more V is much greater than H.

Actual numbers were published by GALITZIN ('Lectures on Seismometry') and are included in Table III., wherein $\bar{e}$ observed is given by $\tan \bar{e} = V/H$, V and H being directly observed.

The agreement of theory and experiment is good at about 3000 km. and again at 8500 km., and beyond, but a discordant feature is shown in the intermediate range. $\bar{e}$ as calculated continues to increase, but $\bar{e}$ as actually observed falls to a minimum at 4000 km. substantially less than the calculated value.

Now the general effect of the crust might be expected to make $\bar{e}$ greater than the value calculated from the time curves. It is extremely difficult to account for the discrepancy actually observed. In view of the exhaustive tests applied by GALITZIN to his instruments, I do not think the discrepancy can be attributed to instrumental error. Many more observations have since been made at Pulkovo, and the results will be awaited with interest. We shall also require observations at other stations before we can decide whether the effect is peculiar to Pulkovo or characteristic of

Table III.

Epicentral distance. Δ in kilometres.	For P.			For S. e from time curve.
	e from time curve.	$\bar{e}$ computed.	$\bar{e}$ observed at Pulkovo.	
	°	°	°	°
0	0	22	—	0
500	11	23	—	12
1,000	21	27	—	22
1,500	30	32	—	30
2,000	37	37	—	37
2,500	44	42	48	43
3,000	49	47	44	49
3,500	53	52	43	54
4,000	57	54	42	57
4,500	60	58	43	60
5,000	63	60	44	62
5,500	65	62	46	63
6,000	65	62	48	63
6,500	65	63	51	63
7,000	65	63	54	64
7,500	66	63	58	64
8,000	66	64	62	65
8,500	67	64	65	65
9,000	67	65	67	66
9,500	68	66	68	67
10,000	69	67	70	67
10,500	70	67	71	68
11,000	70	68	72	69
11,500	71	69	72	70
12,000	72	70	73	71
12,500	73	71	73	72
13,000	74	72	74	73

the earth as a whole. The explanation will then have to be sought either in some structural peculiarity of the rocks composing the crust at Pulkovo or else in a modification of the general time curves. But until the required data are available we must be content with recording a discrepancy that requires further examination in the future.

The first impulse P which arrives by a path of minimum time must be regarded as the only part of a seismogram that arises from one definite cause. Immediately after P the seismogram is due to a variety of causes. Thus in discussing any subsequent pronounced effects that occur it is important to keep in view that they are superimposed on general disturbance which we do not for the moment seek to disentangle.

The relative magnitude of a reflected longitudinal disturbance PR_n that has been reflected n times at the surface to that of the first impulse P may be regarded as determined by two main circumstances: (1) As a disturbance proceeds outwards from the focus it undergoes reduction of amplitude by spherical divergence and general scattering as the wave proceeds through the earth; (2) at each reflexion a sudden change of amplitude takes place.* The first factor appears to be much too difficult to deal with at present. A tentative law of diminution of amplitude suggests itself, viz., $S^{-1}e^{-kS}$, where S is the length of the path. The factor would then be $\frac{S}{S_n}e^{-k(S_n-S)}$. The numerical labour of testing this is rather serious, and from some rough trials I do not think it contains much promise of explaining the observations.

Accordingly, I pass to the second factor, which is within our powers and which goes a considerable way to explaining the facts. In so far as it fails it may give a clue to the proper form of the first factor.

Starting with epicentral distance Δ, Table III. give the corresponding angle e and fig. 2 the values of H and V for the first impulse P. For PR_n we divide Δ into $(n+1)$ parts and obtain the value of e corresponding to $\Delta/(n+1)$. Fig. 2 then gives the corresponding values of ${}_nA_2$, H_n and V_n. Thus the horizontal and vertical components of motion where PR_n meets the surface are $({}_nA_2)^n H_n$ and $({}_nA_2)^n V_n$.

Table IV. shows the results obtained in this way for different distance Δ, and between each column of magnitudes the time interval between the disturbances has

TABLE IV.

Epicentral distance Δ in kilometres.		P.	Time difference.	PR_1.	Time difference.	PR_2.	Time difference.	PR_3.	Time difference.	PR_4.
			seconds.		seconds.		seconds.		seconds.	
1,000	H	1·62	2	0·25	1	0·12	0	0·076	0	0·045
	V	0·83		0·086		0·047		0·031		0·021
2,000	H	1·60	15	0·084	3	0·018	1	0·0091	1	0·011
	V	1·20		0·043		0·0080		0·0039		0·0049
3,000	H	1·38	40	0·17	10	0·0044	4	0·00078	1	0·00040
	V	1·48		0·094		0·0022		0·00036		0·00018
6,000	H	0·93	144	0·59	55	0·067	25	0·0018	9	0·000023
	V	1·79		0·64		0·050		0·0011		0·000013
9,000	H	0·86	213	0·71	118	0·26	62	0·032	24	0·00094
	V	1·82		1·12		0·27		0·027		0·00065
12,000	H	0·68	256	0·70	182	0·42	106	0·11	68	0·014
	V	1·89		1·35		0·59		0·12		0·012

been inserted. It should be understood that for each distance the primary incident disturbance is taken as unity.

* It may even be the case that disturbances travelling in different directions from the focus have different magnitudes.

Only the first four reflexions are computed, and even this is rather far for the short distances. The time curve is not sufficiently known for short distances and, moreover, the effect of finite depth of focus would come in.

We first note that successive reflexions differ in sign. The table shows that for distances up to 3000 km. the reflected effects are small in comparison with P, but as we pass to 6000 km. PR_1 becomes about two-thirds of P in the horizontal component and about one-third in the vertical component. As we pass to 12,000 km. PR_1 actually exceeds P in the horizontal component and is only somewhat less in the vertical component. Moreover, the second reflection, and even the third, are not insignificant.

The table also shows that when the reflected wave has angle e about 20° there is a large drop in the magnitude. As in the case of 2000 km., the successive reflexions show a tendency to increase for a certain range, although they must finally vanish. In this connexion I may refer to a point which I have elsewhere proved, viz., that longitudinal disturbance cannot be propagated along a plane surface. Prof. TURNER, 'B.A. Report,' 1915, remarks that this is in apparent conflict with KNOTT's result, that there is complete reflexion when $e = 0$. There is, however, no conflict, for although the reflected amplitude is then exactly equal to the incident amplitude, there is a change of sign, so that the two disturbances exactly annul and there is no resultant motion at all.

We turn now to the experimental evidence. Reflected waves have never been recognised as distinct features for Δ less than 3000 km. I may, however, draw attention to two records ('Modern Seismology') of Turkish earthquakes Δ about 2500 km. and a Pulkovo record of an Icelandic earthquake Δ about 2500 km. (July 26, 1913). These all show somewhat similar disturbance within the 40 seconds succeeding P, and raise the question whether the whole series of reflexions which must occur within this time could have combined to affect the seismogram measurably.

When we reach Δ from 6000 km. to 9000 km., PR_1 becomes a marked feature comparable with P itself, and PR_2 only slightly less so, but PR_3 and the succeeding reflexions are not clearly made out. Specimen records, figs. 4 and 5 (Plates 3 and 4), to which we shall return, exhibit this. Passing to Δ from 10,000 km. to 12,000 km. we can usually trace the reflexions a little further, and PR_1 may be two or even three times the magnitude of P.

Thus the numbers in Table IV. are in remarkably good general agreement with observation, showing how PR_1 acquires increasing importance as Δ increases. But although the table goes in the right direction it does not go far enough. It removes, however, the main difficulty in understanding the variations, and suggests the necessity of investigating reflexion from a variable layer instead of an ideal surface, and also of considering the effect of spherical divergence. The traces, figs. 4 and 5, are photographically reproduced from those issued by GALITZIN in 1914, but are reduced to $\frac{5}{6}$ original size. The lettering of the reflexion is the Pulkovo judgment, and I have added arrows to show the theoretical times.

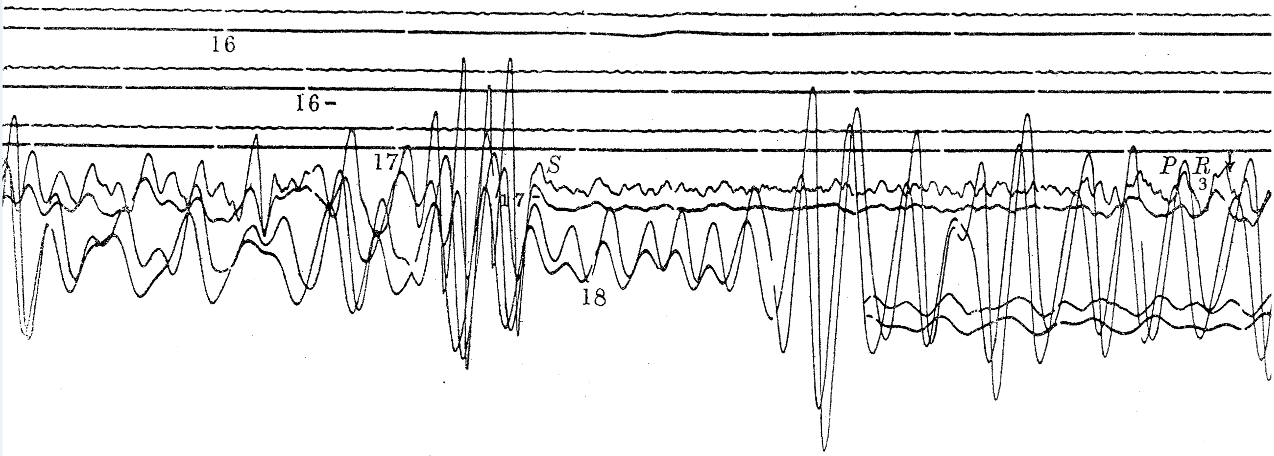
We note how sharp P is in fig. 4 as compared with fig. 5. PR_1 is a quite clear feature in both, although it arrives earlier than the theoretical time by 30 to 40 seconds. In fig. 4 PR_1 is still sharp in the horizontal components but smoother in the vertical. It is distinctly smaller than P and clearly of opposite sign. In fig. 5 PR_1 is smooth in all components, and only slightly less than P. In this case, also, it is of opposite sign to P. In fig. 4 PR_2 arrives about the theoretical time. It is smoother than P in all components, about the same magnitude as PR_1, and of the same sign as P. In fig. 5 PR_2 is apparently early, is about the same magnitude as PR_1, and I think of the same sign as PR_1. In fig. 4 PR_3 is about right as regards time, but is a somewhat vague movement, and PR_4 is absent. In fig. 5 PR_3 and PR_4 marked as earlier than the theoretical time are, in my opinion, too insignificant to merit consideration. Thus theory agrees in some respects and differs in others.

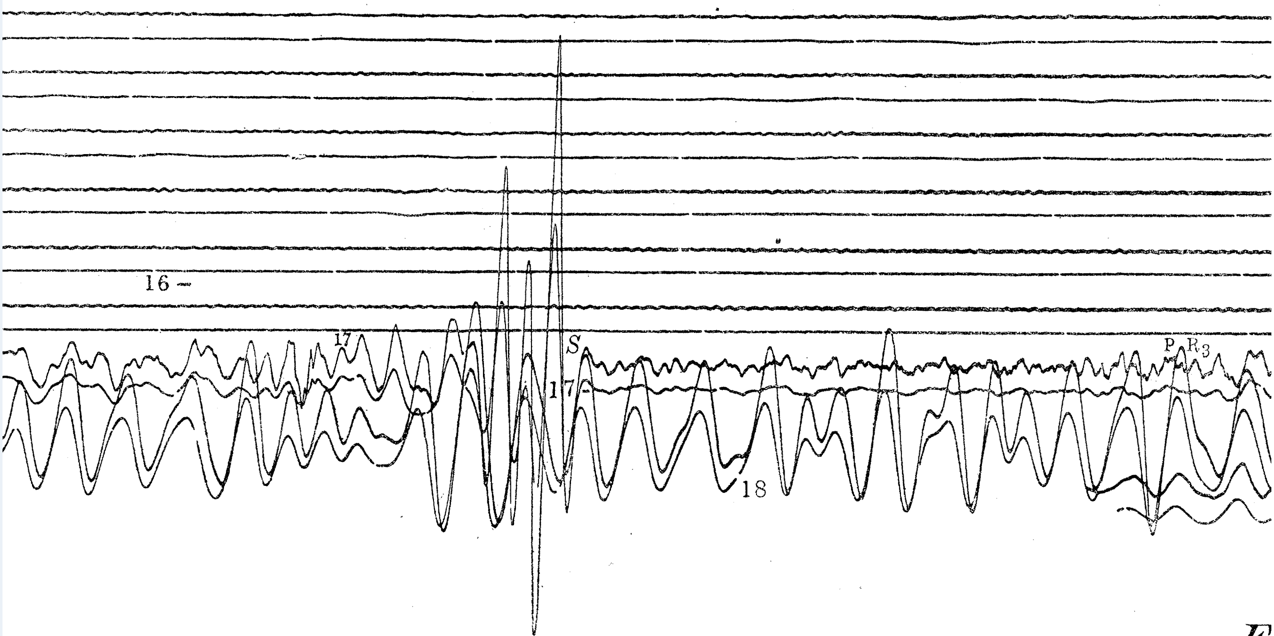
The appearance of P and its reflexions in fig. 5 is strongly suggestive of a limited train following an impulse, and the same may be said of PR_2 and PR_3 in fig. 4.

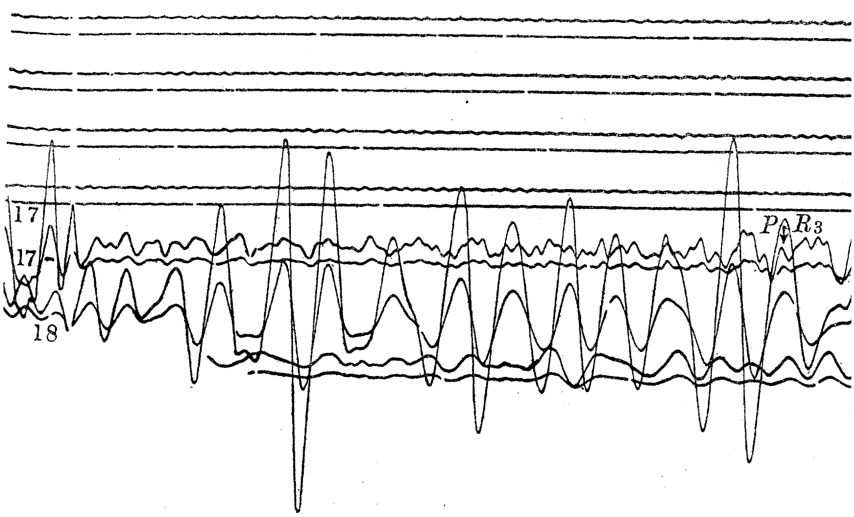
As we have shown, such an effect (established by LAMB for water waves) might be produced by dispersion or reflexion from a variable layer, my own view being that dispersion is not of much importance in seismology. On either hypothesis, however, a sharp impulse may on reflexion give rise to a limited train of periodic waves of distinctly larger amplitude than the primary impulse without violating the principle of energy. Everything depends on the sharpness with which the impulse rises to its maximum (*e.g.*, 1 second) and the number and period of the waves set up. Such an effect may be a contributing factor in the apparent magnitude of the reflexions as compared with P.

The discrepancy in time between PR_1 and the theoretical value obtained from the time curve for P is not a mere accident in the two cases shown. A large number of others will be found in the Pulkovo 'Bulletins.' From an inspection of these it appears on average that at 6000 km. PR_1 is some 10 seconds early, at 8000 km. about 30 seconds, at 10,000 km. it is correct, while at 12,000 km. it is about 10 seconds late.

In conjunction with GALITZIN'S observed values of the emergence angle $\bar{e}$, we get confirmation that the primary curve for P, as given by ZÖPPRITZ, requires revision. The change required is a depression of the ordinates of the time curve extending from 2000 km. to 6000 km. I mention this because, when the time curves are revised, I think it would be extremely unwise to make any alteration without considering the valuable information revealed by the times for the reflected waves and the apparent angle of emergence $\bar{e}$ by direct measurement. The last members of the P reflexions, although I do not think they can really acquire any importance,







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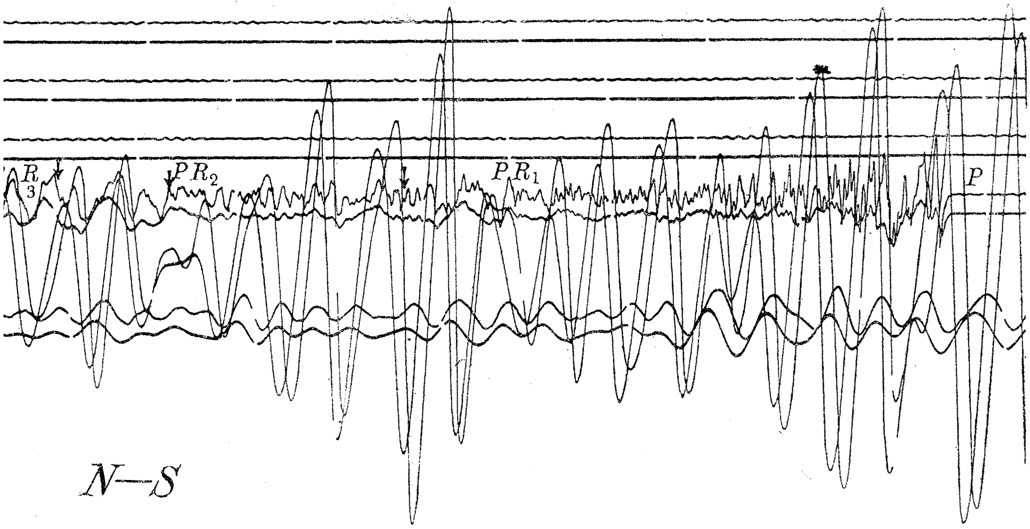
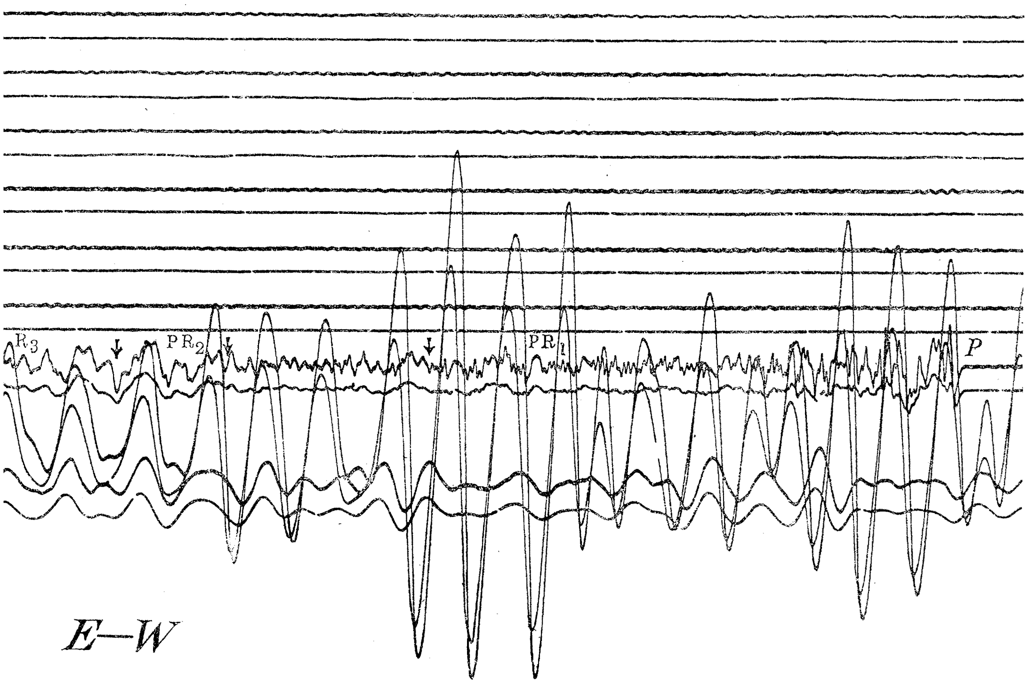


N—S



E—W

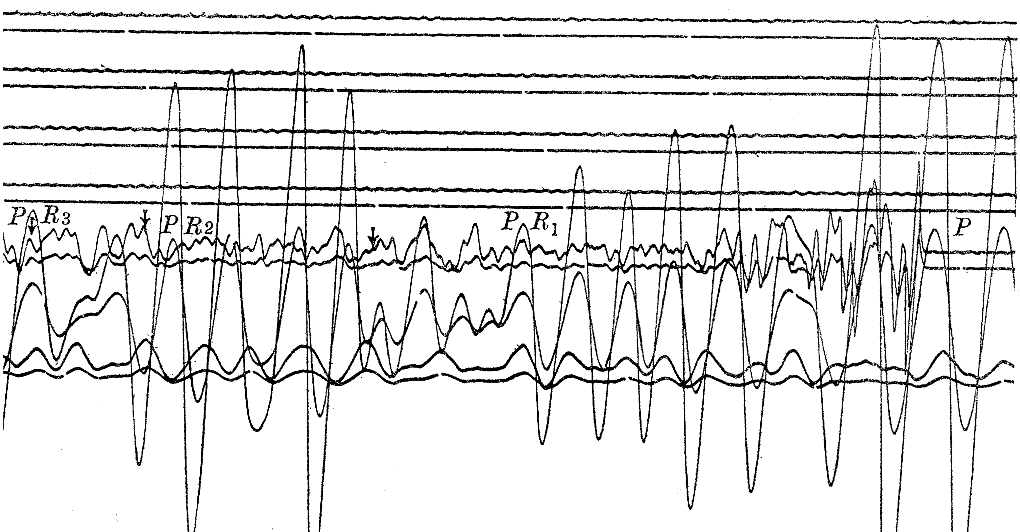


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Fig. 4. Pulkovo seismogram, August 1

Deduced epicentre—Δ = 7100 km., ϕ = 47° N

Note.—Pulkovo recorded each component on two scales of magnification

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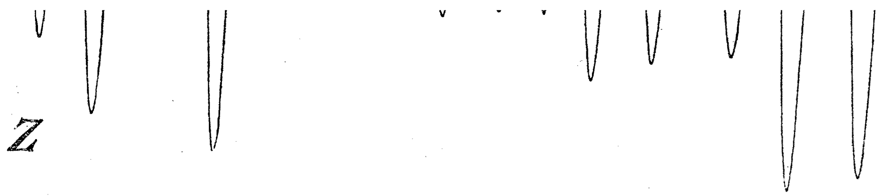

August 1, 1913.
$= 47°$ N., $\lambda = 155°$ E.
nification. Time breaks at one minute intervals.

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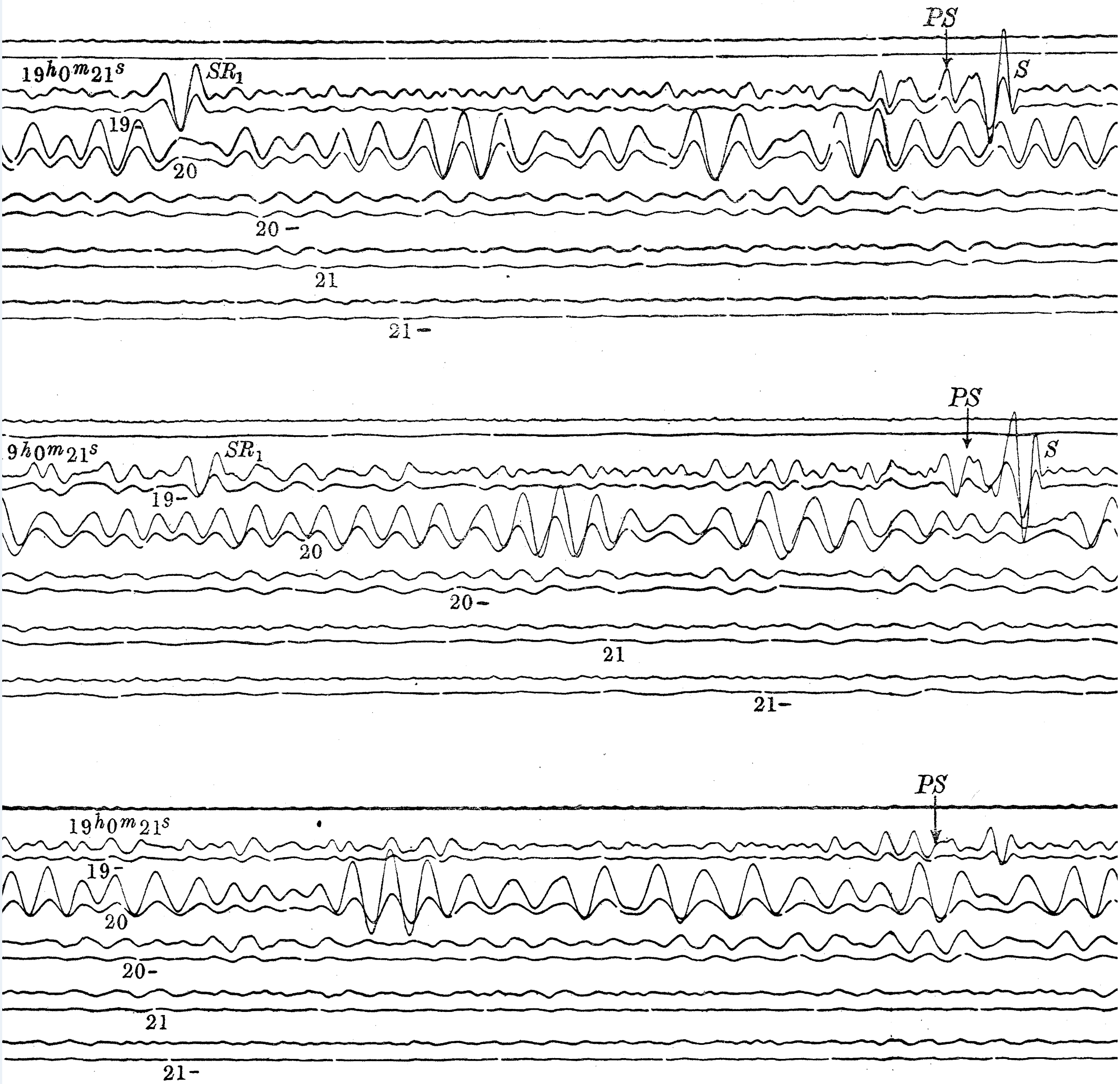

Fi

Deduced

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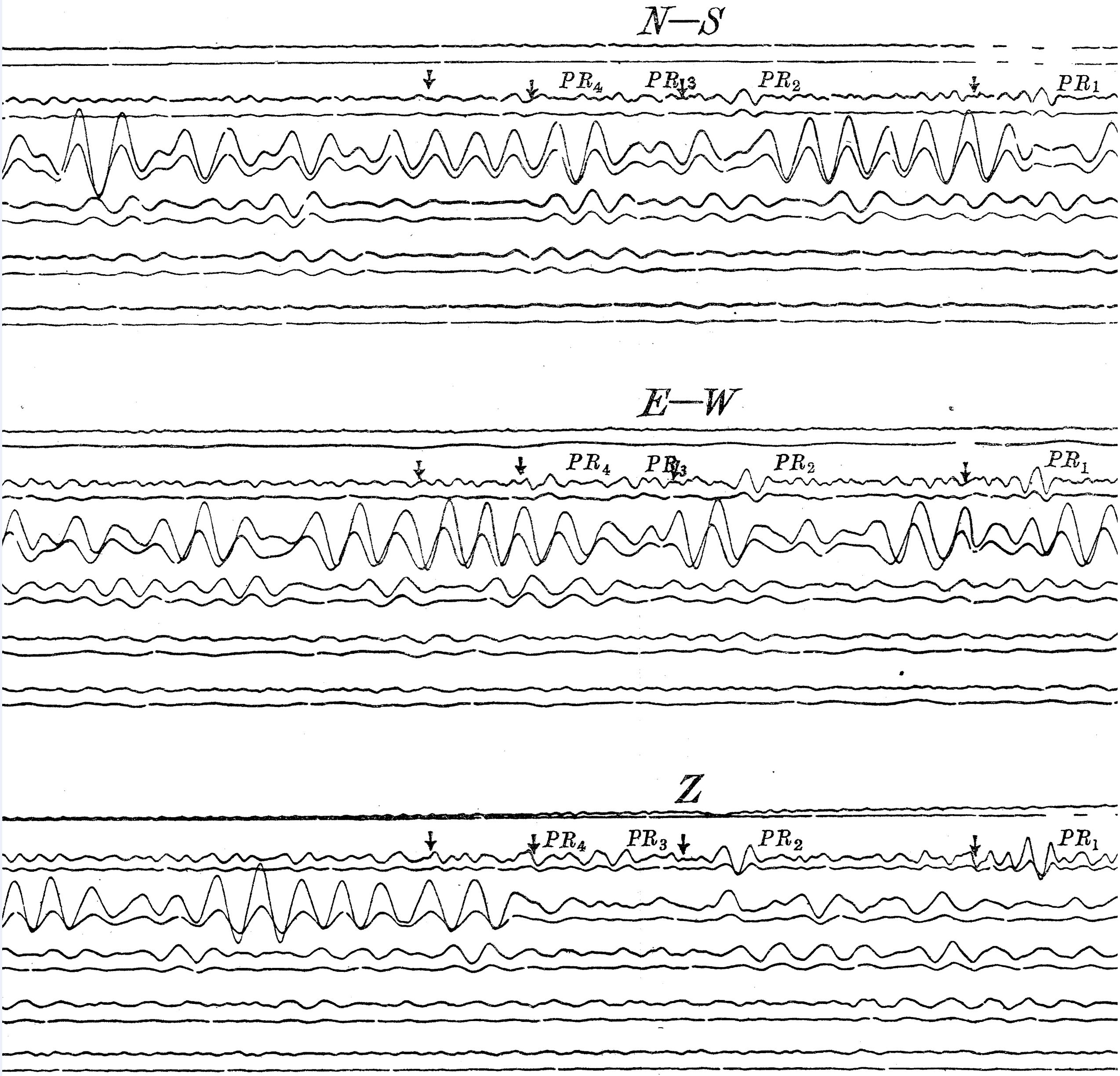

Fig. 5. Pulkovo seismogram, August 15, 1913.
ed epicentre—$\Delta = 8540$ km., $\phi = 27°$ N., $\lambda = 142°$ E.

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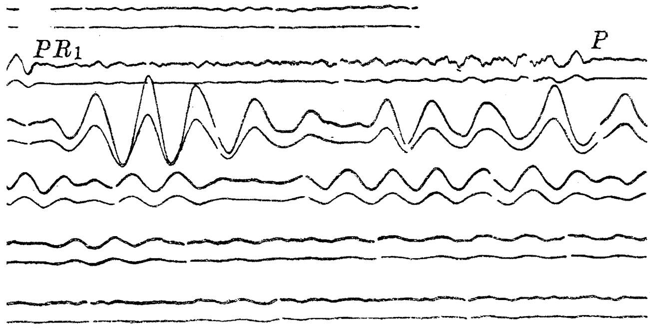
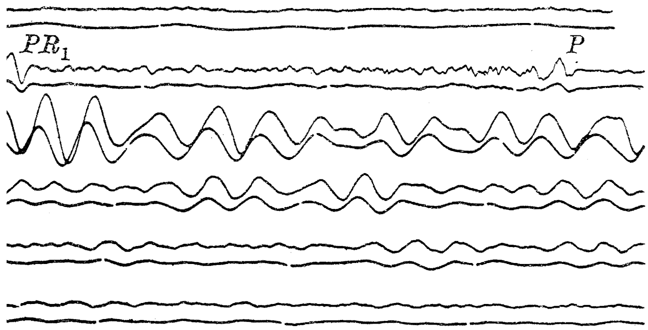
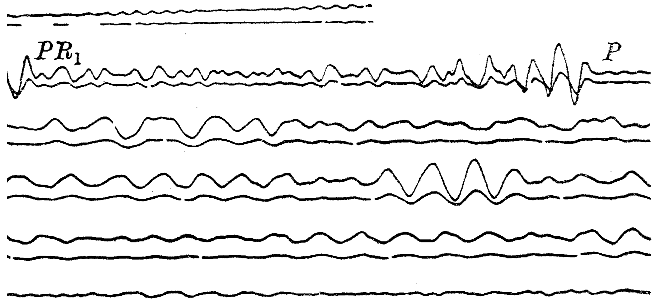

PR1
P


PR1
P


PR1
P

arrive before S until Δ is 11,000 km., but for Δ 11,500 km. and more they arrive after S (according to ZÖPPRITZ'S curve).

We now pass to consideration of the second phase. If we regard the start of the second phase S as representing the arrival of transversal waves we must resolve the disturbance into two: (1) transversal waves with direction of vibration at right angles to the plane containing the station, epicentre and earth's centre, which I will call the azimuth plane; (2) transversal waves with vibration in the azimuth plane.

For the waves vibrating at right angles to the azimuth plane, the vibration is entirely horizontal, there is no vertical component and the waves are reflected without change.

For the waves vibrating in the azimuth plane the matter is more complicated and depends on the angle of impingence. For angles up to 56°, *i.e.*, for Δ about 4000 km., the reflexion is complex. There will be a reflected transversal wave, but it may differ in type from the incident disturbance. There is no true reflected longitudinal wave, but a disturbance confined to the surface exists. I think there is little doubt that this latter disturbance is an important factor in the generation of "Rayleigh" waves. Fig. 3 shows the "modulus" of the disturbance in dotted lines. We cannot speak of an angle of emergence for this region, but we note that the modulus of the reflected transversal wave remains constant and = 1 from $e = 0°$ up to $e = 56°$. The horizontal motion modulus is 0 at $e = 0°$, rises to a maximum at $e = 22°$, and falls again to zero at $e = 45°$, thereafter it rises rapidly to a value of 4·42 at $e = 56°$. The vertical motion modulus, always greater in this range than that of H, reaches a maximum of 1·52 at $e = 48°$ and then falls to 0 at 56°.

As soon as we pass $e = 56°$ there is a longitudinal as well as a transversal reflected disturbance without change of type. A_2 falls rapidly to 0·05 as e increases to 58°, and thereafter increases to 1 as e passes to 90°. A_3 falls from 2·77 at $e = 56°$ to 0 at 90°. The horizontal movement falls from 4·42 at $e = 56°$ to about 1·8 at $e = 63°$ and recovers to 2 at $e = 90°$. The vertical component rises to 0·92 at $e = 60°$ and then falls to 0 at $e = 90°$. We may note that V is always less than H in this range, the least value of H/V being just under 2 at $e = 63°$.

The very rapid change in value of V and H from $e = 45°$ to $e = 65°$, or Δ from 2500 km. to 5500 km., is most remarkable, and suggests the necessity for a very thorough examination of records for such distances.

S is often very far from sharp, and even in the cases where there is a sharp S, it rapidly develops into a highly periodic disturbance. A well-known feature is the comparatively small amount of vertical disturbance in S. We now find good reason for this.

Little has been done in analysing S from actual records, and our results show how difficult the matter is without the guiding information provided by the phenomena of reflexion. GALITZIN made some analyses of S in his book, but he assumed that the transversal wave arrived at the same angle as P. We have, however, a direct means of examining this point by means of the vertical component. The following examples, taken from specimens published by GALITZIN, illustrate the matter :—

(1) Fig. 5. August 15, 1913. α = azimuth = 58° 37′ N.E. Δ = 8540 km. Epicentre 27° N., 142° E.

Second phase sharp in E.

Movement South = 0. East = 7·1 mm. Vertical = 2·0 mm.

Thus along α we get 6·1 mm. $\perp$ to α 3·7 mm. So that H/V = 3·0, and from the curve, fig. 3, e = about 57° (or 73°).

(2) Fig. 4. August 1, 1913. α = 38° 24′ N.E. Δ = 7100 km. Epicentre 47° N., 155° E. Second phase sharp in E.

North = 3·0 mm. West = 23·0 mm. Vertical = 6·0 mm.

Therefore along α we get 11·9 mm. $\perp$ to α 19·9 mm. Hence H/V = 2, or e about 60°.

(3) July 26, 1913. α = 49° 26′ N.W. Δ = 2490 km. Epicentre 67°·5 N., 18·6 W.

Second phase sharp in E.

South = 9·5 mm. West = 11·0 mm. Vertical = 2·0 mm.

Therefore along α 2·2 mm. $\perp$ to α 14·4 mm. Therefore H/V = 1·1, or e is $< 56°$, and, perhaps, about 53°.

(4) January 13, 1915. α = 37° 21′ S.W. Δ = 2280 km. Epicentre 42° 0′ N., 13° 42′ E.

Second phase sharp in E.

South = 15 mm. West = 30 mm. Vertical = 66 mm.

Therefore along α 30·1 mm. $\perp$ to α 14·8 mm. H/V about 0·5. Therefore e is $< 56°$, about 50°.

These cases are in general agreement with the results of reflexion theory and encourage further examination. Great care has to be exercised in getting precisely corresponding pure movements in all three components, as within even a few seconds the resultant vibration changes rapidly and the peaks do not precisely correspond in all components.

Case (1), fig. 5, illustrates this point very well. The first movement in S to East (which agrees precisely in time with the vertical motion up) reaches its maximum just as the North-south component begins to move. This second movement is first a little to South, then to North, the maximum throw to North coinciding in time with the maximum throw to West and the maximum throw down. Only a very open time scale shows this, and with a contracted scale we should probably have associated the first movements to East and to South together.

In case (1) the second movement of S, which follows the first movement immediately, is South = 3·5 mm., West = 8·4 mm., Vertical = 3·0 mm. This gives

9·0 mm. along α and 1·4 mm. at right angles to α and H/V = 3, the same as in the first movement. The angle of impingence deduced is 57° or 73°, the ambiguity arising from the fact that in this region of reflexion H/V has the same value for two different angles. We cannot settle the ambiguity except by observations for different distances, and the fact that case (2) gives e close to 60° (the minimum value of H/V being about 2 at this point), suggests that the larger angle 73° is probably the right one. The value from the time curve is 65°, and the discrepancy is of importance. I consider the test by direct observation of the angle a very searching one, and likely to throw considerable light on the question of the accuracy of the time curves.

Passing to the reflexion of transversal waves SR_n, we have to observe that when the vibration is perpendicular to the azimuth plane no change is introduced by reflexion, so that such waves proceed as before. The vibration in the azimuth plane undergoes change in type for Δ less than 4000 km., and at greater distances change of magnitude and change of sign. This introduces very profound changes in the character of the effects to be expected.

In Table V. I have calculated the relative magnitudes of S and SR_1 to be expected on account of reflexion alone and for the waves vibrating in the azimuth plane.

TABLE V.

Δ in kilometres.		Vibration in azimuth plane. S.	Time interval in seconds.	SR_1.
1,000	H	0·52	4	0·46
	V	1·0		0·79
2,000	H	0·36	28	0·52
	V	1·22		1·0
3,000	H	0·50	71	0·48
	V	1·52		1·1
4,000	H	2·3	131	0·36
	V	0·82		1·22
5,000	H	1·8	197	0·16
	V	0·92		1·36
6,000	H	1·78	254	0·50
	V	0·90		1·52
7,000	H	1·78	298	1·62
	V	0·88		0·76
8,000	H	1·78	329	0·28
	V	0·86		0·10
9,000	H	1·80	354	0·13
	V	0·84		0·06
10,000	H	1·80	373	0·25
	V	0·82		0·13
11,000	H	1·83	397	0·32
	V	0·76		0·16
12,000	H	1·85	427	0·32
	V	0·72		0·16

I have enclosed in thick lines the region within which the reflexion is complex and the result probably untrustworthy. I have used the values of e obtained from the time curve, so that the table is only of value in showing the large changes that may be expected. My hope is that the apparent complexity of the observed results will no longer discourage the investigator, but, on the contrary, will yield to careful treatment and lead to results of great value in seismological theory.

Case (1), fig. 5, is such a remarkably clear record and so free from ambiguity that I venture to add a discussion of the SR_1 shown in it. SR_1 is quite clear in the horizontal component, but practically negligible in the vertical component. We find, however, that SR_1 is about 40 seconds earlier than the theoretical time calculated from the time curves. This is also what we found for PR_1. Thus while the time

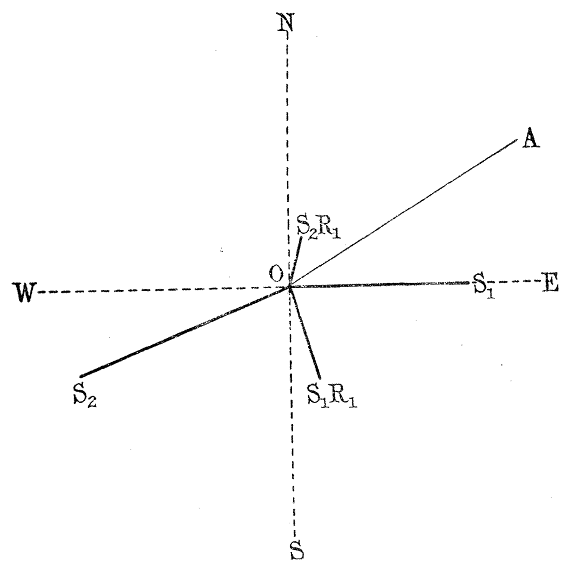


Fig. 6.

interval S−P is not sensibly in error, in my opinion at least, up to 10,000 km., I think that several independent lines of reasoning suggest that the time curves for P and S should be sensibly depressed from about 2500 km. up to 6000 km.

The first sharp movement is entirely to North, and this movement attains its maximum before the East-west component begins to move. We thus enquire whether this movement in SR_1 is related quantitatively to any movement in S. Proceeding as formerly to ascertain the effects due to reflexion of waves which would otherwise produce the same results at the observation station, I have drawn in fig. 6 the conclusions at which we should arrive.

OS_1 and OS_2 represent the first two movements in S, and OS_1R_1, OS_2R_1 are the corresponding effects to be expected in SR. They are obtained as follows: Resolve OS_1 or OS_2 into its components in and at right angles to the azimuth plane OA.

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The former component suffers no reduction by reflexion, but the latter is altered. The amount depends on what are the correct impingence angles for S and SR. There is room for considerable latitude in this, but I have taken provisionally 73° for S and 57° for SR. With these values the component in the azimuth plane suffers reduction in the proportion 6·7 in H and 6·6 in V, and the sign is reversed. We thus get OS_1R_1, OS_2R_1 for the expected horizontal movement, while the vertical movement is very small indeed.

While the results are thus horizontal movements in the North-south component, they do not agree with the observed movement in SR. S_1R_1 is of the opposite sign and S_2R_1 is too small. While the result is perhaps disappointing, I am still convinced that, reasoning conversely, the systematic analysis of the observed movement of the SR for a number of cases is certain to lead to valuable information as to the circumstances of reflexion, even if it involves a change of our ideas as to the full meaning of these reflexions.

At considerable epicentral distances theory indicates that the general effect of reflexion is to suppress rapidly the component in the azimuth plane and make the SR_n's more truly at right angles to the azimuth plane. But there is an important difference between these successive reflexions and the PR_n's. These tail off to nothing, while the transversal waves, as soon as the reflexion takes place at angles less than 56°, may be expected to undergo a change of type rather than of magnitude. Thus the tail of the transversal waves merges into the start of the long wave phase as observation undoubtedly shows to be the case.

In my opinion the marked periodicity of the transverse reflexions indicates more clearly than do the longitudinal reflexions the necessity for examining the effects of reflexion from a variable layer of thickness comparable with the geological crust.

The present discussion would be incomplete without some reference to waves that have been changed from transversal to longitudinal, or *vice versâ*, on reflexion. These waves were called *Wechselwellen* by Wiechert. The vibration must be in the azimuth plane, and they cannot show as a distinct phenomenon until the distance exceeds that for which the angle of impingence for transversal waves in the azimuth plane exceeds 56°. From the time curves this would be about 4000 km., but Galitzin's direct observations of $\bar{e}$ would make the distance over 7000 km.

These Wechselwellen must be made up of two parts: (1) transversal waves starting from the origin and reaching the station as longitudinal; (2) longitudinal waves starting from the origin and reaching the station as transversal. We should naturally seek to compare the first with the S waves and the second with the P waves, while there is no necessary connexion between the two except that they arrive together, and the vibration should be in the azimuth plane. It must, of course, be understood that in making any such comparisons we assume what is not at all likely to be the case, viz., that waves starting from the origin are symmetrical about the origin. We are, in fact, simply estimating the changes due to reflexion without considering the other circumstances that may lead to differences in the relative magnitude of P and S and their reflexions.

We may best illustrate the possibilities by taking the theoretical case 8500 km., which corresponds closely to fig. 5. The theoretical angle of impingence is 67°, and we find that the PS or SP wave divides the arc so that the longitudinal waves correspond to 2000 km. angle $e = 37°$ and the transversal to 6500 km. angle $e = 64°$, while PS arrives 40 seconds after S.

Taking unit amplitude in each case, we have the following relative values for H and V :—

	H.	V.
P	−0·86	−1·82
PS	+1·9	−0·94
S	−1·8	+0·82
SP	−1·4	−1·06

The signs are important. The absolute sign of V is determined by the actual record and may be + or − for either P or S. PS and SP theoretically arrive together and interfere, but it is likely that they may not coincide in time exactly in practice. However that may be, we see that large relative changes may be expected.

If V is of the same sign for P and S we might have a small V in the Wechselwellen and a large H, while if V is of opposite signs for P and S, the reverse holds. Theoretically, we have the means of separating the two constituents of this phenomenon by means of the vertical component record if the angles of impingence are known.

Thus if A and B are the proportions of PS and SP present at the station, we have

$$K_1A + K_2B = H, \qquad K_3A - K_4B = V,$$

where H and V are observed at the station, and $K_1 \ldots K_4$ are known numerical factors depending only on the angles of impingence and independent of any other circumstance.

We may have a wave twice reflected, viz., PPS. In the present case it would divide the arc into two of 1500 km., corresponding to longitudinal waves $e = 30°$, and one of 5500 km., corresponding to transversal waves $e = 63°$. It should arrive 28 seconds after PS.

We might carry the argument to multiple reflexions until the arc corresponding to transversal waves is that for which e reaches the critical value 56°. Beyond this the argument must cease. Turning to the case fig. 5, we recognise disturbances following after S, but find it difficult to regard these as Wechselwellen because the vibration is not in the azimuth plane. The clearest movement which satisfies this condition I have marked provisionally as PS. It is South 4·5 mm., West 7·0 mm., vertical 2·5 down. This gives azimuth $57\frac{1}{2}°$ N.E. instead of $58\frac{1}{2}°$ N.E. derived from P, which is really quite good agreement. Taking the theoretical angles of impingence, we get

$$-1 \cdot 79A - 1 \cdot 60B = -8 \cdot 3 = -\sqrt{4^2 \cdot 5 + 7^2 \cdot 0},$$

$$+0 \cdot 88A - 1 \cdot 20B = 2 \cdot 5,$$

whence A = 3·9 mm., B = 0·8 mm., as the amplitudes of PS and SP at incidence at Pulkovo.

I am, however, doubtful about the movement, for while E and V are clear enough the N movement is conspicuously like the return of the seismograph to rest after an earlier excursion, and if so the assumption that this is PS is false.

From examination of a number of records for different distances, both from Pulkovo and Eskdalemuir, I am very much disposed to think that the Wechselwellen are not clearly separated from S until the distance exceeds 9000 km., and that at greater distances one is very apt to mistake them for S itself. Indeed I am convinced that I made the mistake myself. If this view is correct, it raises the question whether the marked periodicity which one associates with S at shorter distances is not really due to the Wechselwellen coming earlier than the present time curves would suggest.

In conclusion, it appears that quantitative analysis of reflexion theory on simple lines removes a number of difficulties that have hitherto attended interpretation of seismograms, and suggests that a systematic examination of both times and magnitudes of P, S, and at least PR, SR, and PS or SP, is likely to afford important information about the primary time curves. It is manifest that horizontal component seismographs alone are quite inadequate.

So far as the present investigation fails to resolve all the difficulties, it suggests the proper lines of further investigation.

These are (1) investigation of the reflexion of waves from a variable layer of thickness comparable with that of the earth's crust; (2) investigation of the propagation of an impulse from a disturbed centre into the earth, in order to gain some idea of the relative magnitude of disturbance in different directions and at different distances. The results even for a uniform earth would be a valuable guide towards the law of absorption of waves, for the determination of which material undoubtedly exists in the observed magnitude of reflected waves.

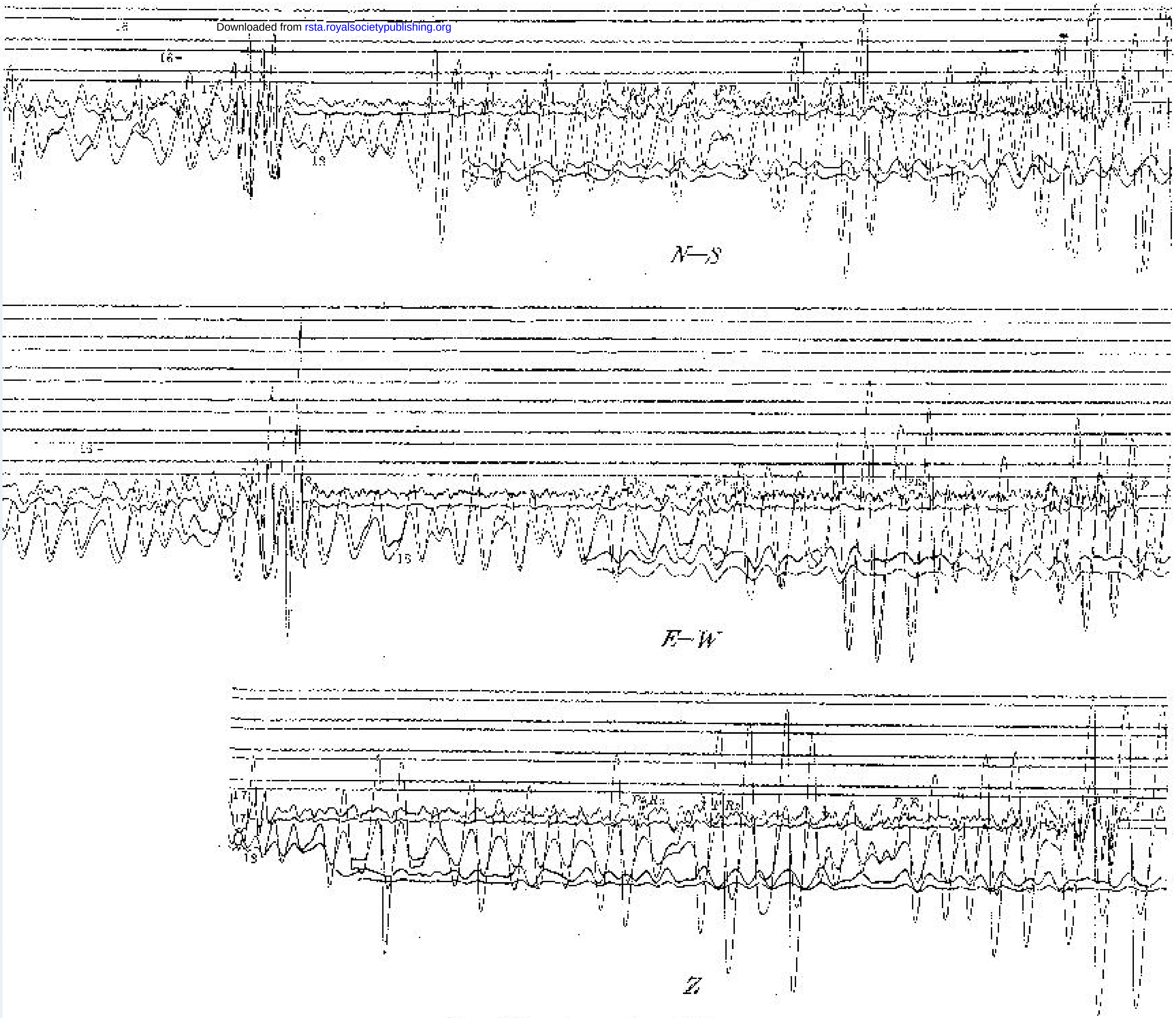

Fig. [illegible].—Pulkovo seismogram, August 2, 1913.

Derived epicentre: $\Delta = 7100$ km., $\phi = 47°$ N., $\lambda = 155°$ E.

Note.—Pulkovo recorded each component on two scales of magnification. Time breaks at one minute intervals.

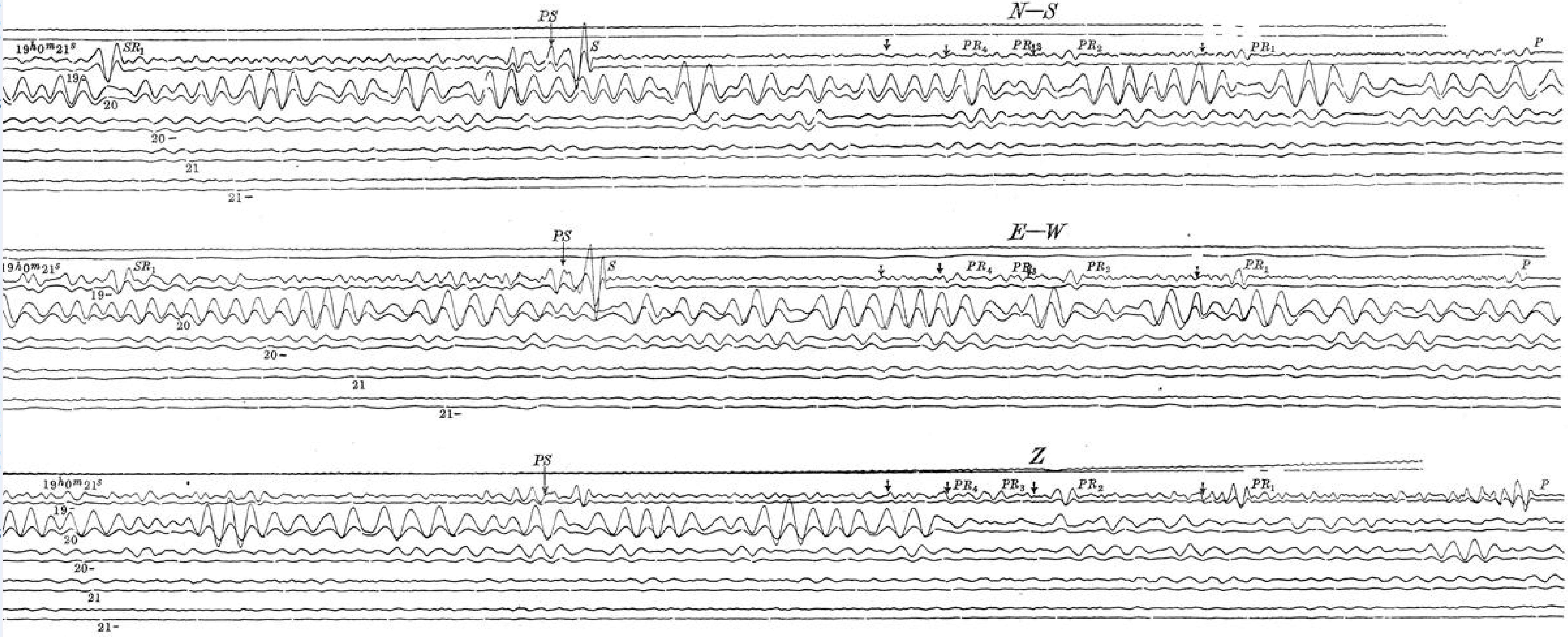

Fig. 5. Pulkovo seismogram, August 15, 1913.
Deduced epicentre—Δ = 8540 km., ϕ = 27° N., λ = 142° E.